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ON PRIME SUBMODULES IN WEAK MULTIPLICATION MODULES

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ABSTRACT
Throughout this article, all rings will be treated as communicative with non zero identity and all modules will be treated as unitary modules. In this paper, some results have been given on prime sub modules in weak multiplication modules. By taking the notion of weak multiplication modules over a commutative ring with identity, we define the notion of product to two submodules of weak multiplication modules and we apply this notion to characterize the prime submodules in weak multiplication modules.

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KEYWORDS: Prime submodules, Multiplication Modules, Weak Multiplication Modules, Product of Weak Multiplication Modules.

INTRODUCTION
Prime submodules of modules have been studied by number of researchers for example Lu. [6], Jenkins and Smith [9] and Mc Casland, Moore and Smith [10] and given many more results on modules. Multiplication module was introduced by Barnad [1] in 1981. Using the notion of prime submodules of modules, the concept of weak multiplication module was developed and many more results have been given by Azizi Shiraz [2]. Some results on prime submodules in multiplication modules were given by Gaur , Maloo and Prakash [3] and Ameri [4] . This paper continues the line of research in the term of weak multiplication modules and we have shown the results using the proof of Gaur, Maloo and Prakash [3] and proof of Ameri [4] which are proved for prime submodules of multiplication modules.

If R is a ring and N is a submodule of an R-module M, the ideal $\{r \in R \mid rM \subseteq N\}$ will be denoted by (N: M) . If N and K are submodules of R-module M then the residual ideal N by K is defined as $(N :_R K) = \{r \in R \mid rK \subseteq N\}$.

Let N be submodule of M and I be an ideal of R the residual submodule N by I is defined as $(N :_M I) = \{m \in M \mid mI \subseteq N\}$. In the special case in which N = 0 the ideal $(0 :_R K)$ is called annihilator of K and it is denoted by $Ann_R(K)$, also the submodule $(0 :_M I)$ is called the annihilator of I in M and it is denoted by $Ann_M(I)$.

PRELIMINARIES
Throughout this paper, R denotes a commutative ring with identity and all related modules are unitary R-modules. The notion of prime submodule in module is an analogous of the notion of prime ideals in ring. Indeed, prime submodules of ring are precisely its prime ideals. (Prime ideals of ring) So, it is most interesting to find out whether some results on prime ideals are also holding for prime sub modules.

In this section we give some basic definitions and results which will be useful to understand the further results.

Definition 2.1 [3] A proper submodule N of an R-module M is said to be prime submodule of R-module M if $ra \in N$ for $r \in R$ and $a \in M$ then either $a \in N$ or $rM \subseteq N$.

(Also see example Lu [6] and Mc Casland and Moore [7])

It is remarkable that if N is a prime submodule of M then P= (N : M) is necessarily a prime ideal of R and therefore N is some time referred as a P-prime submodule of M. (Gaur, Maloo, and Prakash [3]).

Definition 2.2 [1] An R-module M is said to be **multiplication module** if for every submodule N of M , there exists an ideal I of R such that N = IM.

Definition 2.3 [2] An R-module M is called **weak multiplication module** if M does not have any prime submodule or every prime submodule N of M, we have N = IM, where I is an ideal of R.

One can easily show that if an R module M is weak multiplication module then $N = (N :_R M) M$ for every prime submodule N of M (Saymeh [5]) .

MAIN RESULTS

In this section, we obtain some results on prime submodules in the term of weak multiplication module. After that by taking the notion of weak multiplication module over a commutative ring with identity, we define the notion of product of two submodules of a weak multiplication module and we apply this notion to characterize the prime submodules in weak multiplication modules.

Proposition 3.1: Let R be a ring and let M be a finitely generated weak multiplication module such that every prime submodule of M is finitely generated then M is Noetherian.

Proof : Since M is finitely generated weak multiplication module then either if M does not have any prime submodule then the assertion is obvious or every prime submodule N of M, we have N = IM, where I is an ideal of R. In this case, we may take that $M \neq \{0\}$. Then by Gaur, Maloo and Prakash [3] of Theorem 3.2, M is Noetherian.

Now we define the notion of product of two submodules of a weak multiplication module and we apply this notion to characterize the prime submodules in weak multiplication modules.

Definition 3.1: Let M be a weak multiplication R-module and let N be a submodule of M such that N = IM for some ideal I of R, then we say that I is a presentation ideal of N or, for short, I is a presentation of N. Here we denote the set of all presentation ideals of N by $P_r(N)$. It is remarkable, that, it is possible that for a submodule N of a weak multiplication module M, no such presentation ideal exists.

Definition 3.2: Let N and K be two submodules of a weak multiplication module M, such that N = IM and K = JM for some ideal I and J of R then the product of N and K is denoted by N.K or NK is defined by IJM.

Clearly, NK is a submodule of M and contained in $N \cap K$. Now we show that the product of two prime submodules of a weak multiplication module is defining an operation on prime submodules of a weak multiplication module M.

Proposition 3.2 : Let N = IM and K = JM be prime submodules of a weak multiplication R-module M. Then, the product of N and K is independent of presentations of N and K.

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Proof : Let N and K be two prime submodules of a weak multiplication R-module M.

Let $N = I_1M = I_2M = N'$

and $K = J_1M = J_2M = K'$ for ideal I_i and J_i of R, I = 1, 2.

Consider $rsm \in NK = I_1 J_1 M$ for some $r \in I_1$, $s \in J_1$ and $m \in M$.

From $J_1 M = J_2 M$, we have

$$sm = \sum_{i=1}^{n} r_i m_i , \quad \text{for} \quad r_i \in J_2 , \; m_i \in M.$$

Then

$$rsm = \sum_{i=1}^{n} r_i (r m_i) \qquad \ldots(1)$$

From $r m_i \in I_1 M = I_2 M$, we conclude that $r m_i = \sum_{j=1}^{k} t_{ij} m'_{ij}$, for $t_{ij} \in I_2$, $m'_{ij} \in M$ ...(2)

Thus from (1) and (2) we get,

$$rsm = \sum_{i=1}^{n} \sum_{j=1}^{k} r_i t_{ij} m'_{ij}$$

Therefore

$$rsm \in I_2 J_2 M$$

and hence

$$I_1 J_1 M \subseteq I_2 J_2 M.$$

Similarly, we have

$$I_2J_2M \subseteq I_1J_1M.$$

Therefore $I_1J_1M = I_2 J_2M$.

Hence, the product of two prime submodules N and K of a weak multiplication R- module M, is independent of presentations of N and K. Hence, proved the proposition.

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